On k-11-representable graphs

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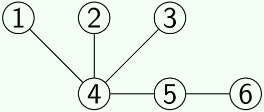
Representation of graphs by words

Basic idea

A **motivation** to study various encodings of graphs by words is the hope, for a given (difficult) problem on graphs, to be able to find a **suitable encoding** that would allow to translate the problem on graphs to an **easier** problem on words, and solve it. Such an encoding does **not** have to be **optimal in size**.

Example: Prüfer codes (sequences) to encode labelled trees (1918)

Provides a proof of **Cayley's formula** (n^{n-2}) to enumerate labelled trees on n vertices.



Remove the leaf with the **smallest label** and record its neighbour:
4445 (the last neighbour does not need to be recorded)

Word-representable graphs

All graphs considered by us are simple (no loops, no multiple edges).

Word-representable graph

A graph $G = (V, E)$ is word-representable if there exists a word w over the alphabet V such that letters x and y, $x \neq y$, alternate in w **if and only if** $xy \in E$. (w **must** contain **each** letter in V)

Word-representant

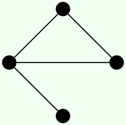
w is a word-representant. We say that w represents G.

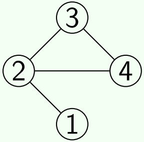
Remark

We deal with **unlabelled graphs**. However, to apply the definition, we need to label graphs. Any labelling of a graph is **equivalent** to any other labelling because letters in w can always be renamed.

Word-representable graphs

Example

The graph  is word-representable.

Indeed, 

can be represented by 1213423.

Remark

The class of word-representable graphs is hereditary. That is, removing a vertex v in a word-representable graph G results in a word-representable graph G'.

The best ways to learn about the subject

WIKIPEDIA
The Free Encyclopedia

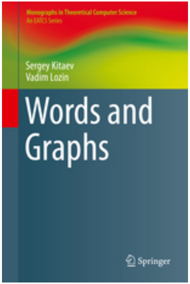
Main page

Article Talk Read

Word-representable graph

From Wikipedia, the free encyclopedia

In the mathematical field of graph theory, a **word-representable graph** is a graph that can be characterized by a word (or se



International Conference on Developments in Language Theory
DLT 2017: Developments in Language Theory pp 36-67 | Cite as

A Comprehensive Introduction to the Theory of Word-Representable Graphs

Authors Authors and affiliations

Sergey Kitaev

Conference paper
First Online: 21 July 2017

5 Citations 396 Downloads

Part of the Lecture Notes in Computer Science book series (LNCS, volume 10396)

Several facts about word-representable graphs

- Introduced by the presenter in **2004** based on the joint research with Steven Seif on the celebrated Perkin's semigroup
- Includes several well-known graph classes, e.g. 3-colorable graphs, circle graphs and comparability graphs
- **Not all** graphs are word-representable. The **minimum** non-word-representable graph is the wheel graph W_5 on 6 vertices
- There is a useful **characterization** of word-representable graphs in terms of certain graph orientations called semi-transitive orientations
- **Recognizing** word-representability is an **NP-complete problem**
- **Enumeration** is known for ≤ 11 vertices (**3 years** of computations!)
- **Word-representants** can be assumed to have the **same number of occurrences of each letter** and be of **length at most** $2n^2$ (for a graph on n vertices)

From word-representation to k-u-representation

No edge between x and y = an occurrence of either xx or yy
= an occurrence of the **pattern 11**

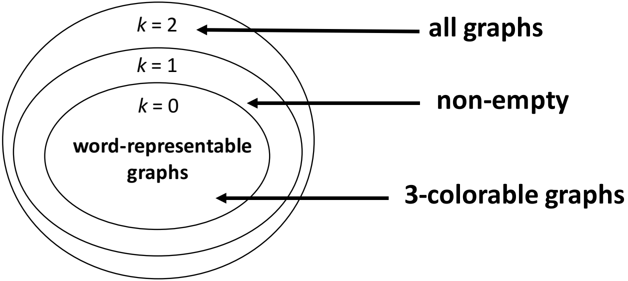
- An idea of Jeff Remmel (2017): Why not to allow a few occurrences of the pattern **11** when defining an edge?
- A graph $G = (V, E)$ is k-11-representable if there exists a word w such that $xy \in E$ **iff** w restricted to x and y contains **at most** k occurrences of the pattern **11**.
- k in the definition of k-11-representable graphs can be thought of as the degree of tolerance
- Note that **0-11-representation** = **word-representability**
- k-u-representation of graphs is defined similarly to the case of $u = 11$ by letting u be any binary pattern

Facts about k-11-representable graphs

The following facts were established in "G.-S. Cheon, J. Kim, M. Kim, S. Kitaev, A. Pyatkin. On k-11-representable graphs. J. of Combin. (2019)"

- k-**11-representability** implies $(k+1)$-**11-representability**
- The class of **0-11-representable graphs** is **strictly** inside of the class of **1-11-representable graphs** (in particular, **all** non-word-representable graphs on up to 7 vertices are 1-11-representable)
- For any graph G, there exists a k such that G is k-**11-representable**
- In fact, **all** graphs are **2-11-representable graphs**
- A graph is an **interval graph iff** it is 1-11-representable by a word containing two copies of each letter; such words characterise **circle graphs** in the 0-11-representation case

The hierarchy related to k-11-representable graphs



Open problem: Can we 1-11-represent **every** graph? In other words, is $k = 1$ equal to $k = 2$?

Hope: The answer to the question is "Yes!", because 2-11-representation of graphs is achieved by a concatenation of permutations (giving just **transitive graphs** in the 0-11-representation case!); however, we can mix letters in words in a more sophisticated way...