2-avoidance

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Permutation Patterns in cyberspace 2020
Interactive video at
`https://uts.h5p.com/content/1291033806121141429`

Definition (2-containment [1,2])

Let σ be a permutation and $F, G \subseteq S^\infty$.

We say that σ 2-*contains* (F, G) if there exists $\gamma <_{\text{subperm}} \sigma$ such that

- $\text{red}(\gamma) \in F$ and
- there is no $\delta <_{\text{subperm}} \sigma$ such that $\gamma <_{\text{subperm}} \delta$ and $\text{red}(\delta) \in G$.

Informally we think of the set G as patterns which can potentially *save* a permutation from being forbidden by F.

Eg: 15234 2-contains $(\{123\}, \{1423\})$

[1] $\text{red}(\alpha)$ — permutation obtained by replacing the ith smallest entry of α by the integer i

[2] S^∞ — set of all reduced permutations

DEFINITION: 2-AVOIDS

A permutation 2-*avoids* (F, G) if it does not 2-contain (F, G). By propositional logic:

Definition (2-avoidance)

Let σ be a permutation and $F, G \subseteq S^\infty$.

σ 2-*avoids* (F, G) if for all $\gamma <_{\text{subperm}} \sigma$, if $\text{red}(\gamma) \in F$ then there exists $\delta <_{\text{subperm}} \sigma$ such that $\gamma <_{\text{subperm}} \delta$ and $\text{red}(\delta) \in G$: δ *saves* γ.

Eg: 15423 2-avoids $(\{123\}, \{1423\})$

We denote the set of all permutations in S^∞ which 2-avoid (F, G) by $\text{Av}_2(F, G)$.

$F = \{3241\}, G = \{41352\}$. Does the perm 2-contain or 2-avoid (F, G)?

- 143562

- 152463

$F = \{1\}, G = \{12, 21\}$. Then $\mathrm{Av}_2(F, G) = S^\infty \setminus \{1\}$.

This shows that the growth of (proper) 2-avoidance sets can be factorial (in contrast to[3])

Question: what growth rates are possible for 2-avoidance?

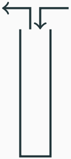
- super-exponential but sub-factorial?
- super-polynomial but sub-exponential? (in contrast to[4])

[3] Marcus and Tardos, “Excluded permutation matrices and the Stanley-Wilf conjecture”, 2004.

[4] Kaiser and Klazar, “On growth rates of closed permutation classes”, 2002/03.

MOTIVATION: 2-PASS POP STACK SORTABLE PERMUTATIONS

[5]



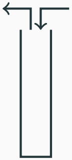
3241

30241

Barred pattern avoidance:

A permutation avoids $4\bar{1}352$ means if it has any subsequence order isomorphic to 3241, that subsequence must be a part of a subsequence order isomorphic to 41352.

[5] deterministic pop stack: sorting device with two operations · push: move a token from the input to the top of the stack · pop: move the entire stack contents to the output · always push unless the token on the top of the stack is smaller in value than the token to be pushed from the input.



3241

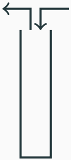
30241

Theorem (Pudwell and Smith[6])

The set of 2-pass pop stack sortable permutations is equal to

$$Av_B\left(\{2341, 3412, 3421, 4123, 4231, 4312, 3241, 4\bar{1}352, 413\bar{5}2\}\right)$$

[6] Pudwell and Smith, "Two-stack-sorting with pop stacks", 2019.



32451

4631572

4731562

If we were to characterise 3-pop stack sortable permutations as those avoiding some list containing $4\bar{6}3\bar{1}572$ and $4\bar{7}3\bar{1}562$, then we would be mistaken:

4731562 does not avoid this list since it fails to avoid $4\bar{6}3\bar{1}572$.

Theorem (Pudwell and Smith)

The set of 2-pass pop stack sortable permutations is equal to

$$Av_B\left(\{2341, 3412, 3421, 4123, 4231, 4312, 3241, 4\bar{1}352, 413\bar{5}2\}\right)$$

becomes

Theorem (Pudwell and Smith)

The set of 2-pass pop stack sortable permutations is equal to

$$Av_2(\{2341, 3412, 3421, 4123, 4231, 4312, 3241, 4132\}, \{41352\})$$

Theorem (E, Goh[7])

For each $k \in \mathbb{N}$ there exist finite sets $F_k, G_k \subseteq S^\infty$ such that σ is sortable by k-passes through a pop stack iff σ 2-avoids (F_k, G_k).

More details in Andrew's talk tomorrow.

This[8] answers Claesson and Guðmundsson's[9] question: is there a

"*useful permutation pattern characterization of the k-pop stack-sortable permutations*"?

[7] Elder and Goh, "k-pop stack sortable permutations and 2-avoidance", 2019.

[8] we think, if you like 2-avoidance

[9] Claesson and Guðmundsson, "Enumerating permutations sortable by k passes through a pop-stack", 2018.

THANKS