Schedules in Square Cases of Social Golfer Problem

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Introduction

Social Golfer Problem (*CSPLib - prob010*)

- Scheduling $n = g * s$ golfers into g groups of s golfers for w weeks so that no two golfers play in the same group more than one week.
- An instance is denoted by $g - s - w$.

- 1998 - sci.op-research (by Bigwind777) - 8-4-w (max w is 10)
- 1782 - Euler's officer problem (6-6-4, not exist)
- 1850 - Kirkman's schoolgirl problem (5-3-7, exist)

Results - $p^r - p^r - (p^r + 1)$

Theorem [*Liu, Löffler, Hofstedt (ICAART 2019)*]

There exist solutions for 7-7-8, 8-8-9, 9-9-10, 11-11-12, 13-13-14, 6-3-8, 6-4-7 and 7-3-10.

- $w(s-1) \leqslant gs - 1$
- $w \leqslant \lfloor \frac{gs-1}{s-1} \rfloor = \lfloor \frac{n^2-1}{n-1} \rfloor = n + 1$, where $n = p^r$

Theorem

There exists a solution for $n - n - (n+1)$ when $n = p^r$, where p is a prime number and r is a positive integer.

Results - $p^r - p^r - (p^r + 1)$

- Permutation Pattern ($g = s = 5$)
- Preparation

	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16	S17	S18	S19	S20	S21	S22	S23	S24	S25
W1	1	1	1	1	1	2	2	2	2	2	3	3	3	3	3	4	4	4	4	4	5	5	5	5	5
W2	1	2	3	4	5																				
W3	1	2	3	4	5																				
W4	1	2	3	4	5																				
W5	1	2	3	4	5																				
W6	1	2	3	4	5																				

- Algorithms
  - Fill Block (W2, S(5k+6)) with 1, where k is a positive integer
  - Fill W2 and S(5k+6) with related circular permutation of 2345
  - Find suitable Block 2
  - Use Block 2 to fill Block 3 to 5 with the same pattern of W2
- Two solutions

	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16	S17	S18	S19	S20	S21	S22	S23	S24	S25
W1	1	1	1	1	1	2	2	2	2	2	3	3	3	3	3	4	4	4	4	4	5	5	5	5	5
W2	1	2	3	4	5	1	2	3	4	5	1	3	4	5	2	1	4	5	2	3	1	5	2	3	4
W3	1	2	3	4	5	2	3	5	1	4	3	4	2	1	5	4	5	3	1	2	5	2	4	1	3
W4	1	2	3	4	5	3	5	4	2	1	4	2	5	3	1	5	3	2	4	1	2	4	3	5	1
W5	1	2	3	4	5	4	1	2	5	3	5	1	3	2	4	2	1	4	3	5	3	1	5	4	2
W6	1	2	3	4	5	5	4	1	3	2	2	5	1	4	3	3	2	1	5	4	4	3	1	2	5

	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16	S17	S18	S19	S20	S21	S22	S23	S24	S25
W1	1	1	1	1	1	2	2	2	2	2	3	3	3	3	3	4	4	4	4	4	5	5	5	5	5
W2	1	2	3	4	5	1	2	3	4	5	1	3	4	5	2	1	4	5	2	3	1	5	2	3	4
W3	1	2	3	4	5	2	5	4	1	3	3	2	5	1	4	4	3	2	1	5	5	4	3	1	2
W4	1	2	3	4	5	3	4	2	5	1	4	5	3	2	1	5	2	4	3	1	2	3	5	4	1
W5	1	2	3	4	5	4	1	5	3	2	5	1	2	4	3	2	1	3	5	4	3	1	4	2	5
W6	1	2	3	4	5	5	3	1	2	4	2	4	1	3	5	3	5	1	4	2	4	2	1	5	3

Results - $p^r - p^r - (p^r + 1)$

- Finite Field $(g = s = 4 = 2^2 \leftrightarrow \mathbb{Z}_2[x]/ < x^2 + x + 1 >)$
- Preparation

	S1	S2	S3	S4
W1	0	0	0	0
W2	0	1	2	3
W3	0	1	2	3
W4	0	1	2	3
W5	0	1	2	3

S5	S6	S7	S8
1	1	1	1

S9	S10	S11	S12
2	2	2	2

S13	S14	S15	S16
3	3	3	3

- Algorithms
  - Use multiplication to decide S(4k+1)
  - Use plus to decide Block 2 to 4.

*	0	1	x	x+1
0	0	0	0	0
1	0	1	x	x+1
x	0	x	x+1	1
x+1	0	x+1	1	x

+	0	1	x	x+1
0	0	1	x	x+1
1	1	0	x+1	x
x	x	x+1	0	1
x+1	x+1	x	1	0

+	0	x	x+1	1
0	0	x	x+1	1
x	x	0	x	x+1
x+1	x+1	1	0	x
1	1	x+1	x	0

+	0	x+1	1	x
0	0	x+1	1	x
x+1	x+1	0	x	1
1	1	x	0	x+1
x	x	1	x+1	0

- Solution

	S1	S2	S3	S4
W1	0	0	0	0
W2	0	1	2	3
W3	0	1	2	3
W4	0	1	2	3
W5	0	1	2	3

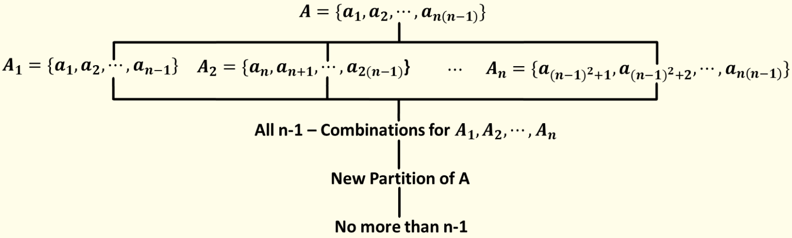
S5	S6	S7	S8
1	1	1	1
0	1	2	3
1	0	3	2
2	3	0	1
3	2	1	0

S9	S10	S11	S12
2	2	2	2
0	2	3	1
2	0	1	3
3	1	0	2
1	3	2	0

S13	S14	S15	S16
3	3	3	3
0	3	1	2
3	0	2	1
1	2	0	3
2	1	3	0

Results - $p^r - (p^r - 1) - p^r$

- $w \leqslant \lfloor \frac{gs-1}{s-1} \rfloor = \lfloor \frac{n(n-1)-1}{n-2} \rfloor = n + 1$, where $n = p^r \geqslant 4$



Theorem

- There exists a solution for $n - (n-1) - n$ when $n = p^r \geqslant 4$, where p is a prime number and r is a positive integer.
- There exists a solution for $3 - 2 - 5$.

Future Works - Uniqueness

Example. Permutation Pattern ($g = s = 5$)

- Solution 1

	S1	S2	S3	S4	S5
W1	1	1	1	1	1
W2	1	2	3	4	5
W3	1	2	3	4	5
W4	1	2	3	4	5
W5	1	2	3	4	5
W6	1	2	3	4	5

	S6	S7	S8	S9	S10
W1	2	2	2	2	2
W2	1	2	3	4	5
W3	2	3	5	1	4
W4	3	5	4	2	1
W5	4	1	2	5	3
W6	5	4	1	3	2

	S11	S12	S13	S14	S15
W1	3	3	3	3	3
W2	1	3	4	5	2
W3	3	4	2	1	5
W4	4	2	5	3	1
W5	5	1	3	2	4
W6	2	5	1	4	3

	S16	S17	S18	S19	S20
W1	4	4	4	4	4
W2	1	4	5	2	3
W3	4	5	3	1	2
W4	5	3	2	4	1
W5	2	1	4	3	5
W6	3	2	1	5	4

	S21	S22	S23	S24	S25
W1	5	5	5	5	5
W2	1	5	2	3	4
W3	5	2	4	1	3
W4	2	4	3	5	1
W5	3	1	5	4	2
W6	4	3	1	2	5

Groups in W2: (1, 6, 11, 16, 21), (2, 7, 15, 19, 23), (3, 8, 12, 20, 24), (4, 9, 13, 17, 25), (5, 10, 14, 18, 22)

- New Solution 2
  - Switch block 3 with block 5 and relabel golfers
  - Switch 3rd golfer with 5th golfer in each block and relabel golfers
  - Switch 4th week with 6th week and relabel week

	T1	T2	T3	T4	T5
V1	1	1	1	1	1
V2	1	2	5	4	3
V3	1	2	5	4	3
V4	1	2	5	4	3
V5	1	2	5	4	3
V6	1	2	5	4	3

	T6	T7	T8	T9	T10
V1	2	2	2	2	2
V2	1	2	5	4	3
V3	2	5	3	1	4
V4	5	3	4	2	1
V5	4	1	2	3	5
V6	3	4	1	5	2

	T11	T12	T13	T14	T15
V1	5	5	5	5	5
V2	1	5	4	3	2
V3	5	4	2	1	3
V4	4	2	3	5	1
V5	3	1	5	2	4
V6	2	3	1	4	5

	T16	T17	T18	T19	T20
V1	4	4	4	4	4
V2	1	4	3	2	5
V3	4	3	5	1	2
V4	3	5	2	4	1
V5	2	1	4	5	3
V6	5	2	1	3	4

	T21	T22	T23	T24	T25
V1	3	3	3	3	3
V2	1	3	2	5	4
V3	3	2	4	1	5
V4	2	4	5	3	1
V5	5	1	3	4	2
V6	4	5	1	2	3

Groups in V2: (1, 6, 11, 16, 21), (2, 7, 15, 19, 23), (5, 10, 14, 18, 22), (4, 9, 13, 17, 25), (3, 8, 12, 20, 24)

- Two solutions have the same structure.

Is the solution unique for $p^r - p^r - (p^r + 1)$?

Future Works - Two Prime Factors

Conjecture

There is no solution for $n - n - (n+1)$ when $pq|n$, where p and q are two different prime numbers.

Theorem

- There is a finite field of order n if and only if $n = p^r$.
- Any two fields with p^r elements are isomorphic.

If there exists a solution for $n - n - (n+1)$, do $\{1, 2, \ldots, n\}$ forms a finite filed?