

Automatic Enumeration of Grid Classes

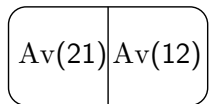
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Joint work with Christian Bean, Jay Pantone and Henning
Ulfarsson

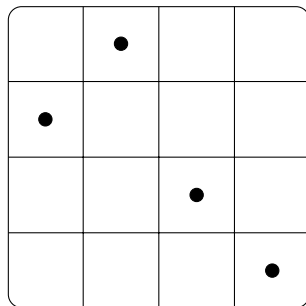
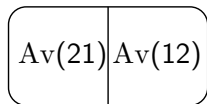
Permutation Patterns 2018



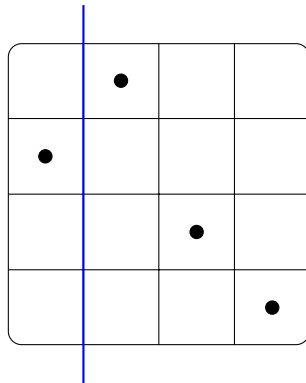
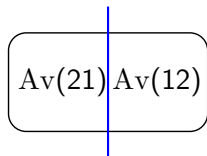
Given a matrix \mathcal{M} whose entries are permutation classes, the permutations in the *grid class* defined by \mathcal{M} , $\text{Grid}(\mathcal{M})$, are those which can have a grid drawn on it such that the subpermutation in each box is in the corresponding permutation class in \mathcal{M} .



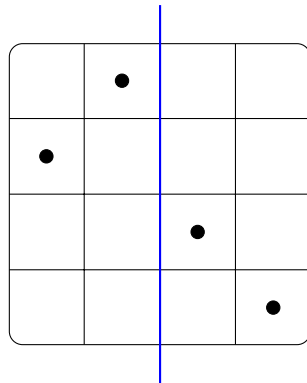
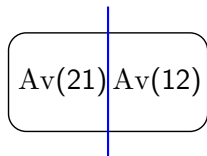
Grid class



Grid class



Grid class



Theorem

Let \mathcal{G} be a $1 \times N$ grid class. There exists a disjoint union of tilings \mathcal{T} such that the gridded permutations in \mathcal{T} are in bijection with the griddable permutations in \mathcal{G} .

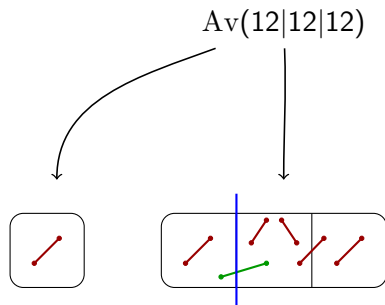
$A_v(12|12|12)$

Disambiguation of a grid class

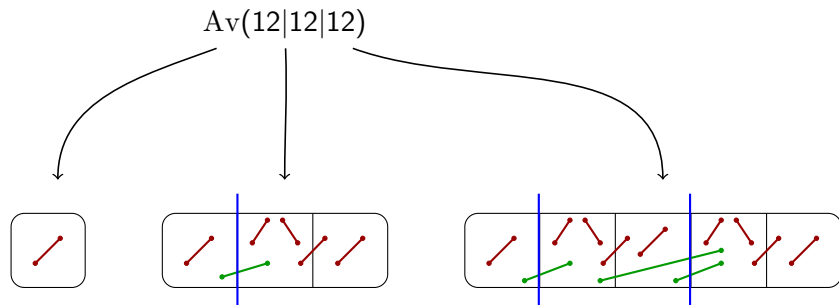
$A_v(12|12|12)$



Disambiguation of a grid class



Disambiguation of a grid class



Disambiguation to combinatorial specification

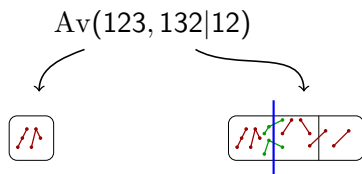
$A_V(123, 132|12)$

Disambiguation to combinatorial specification

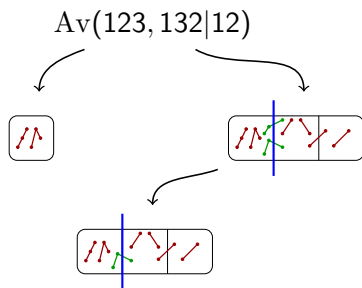
$Av(123, 132|12)$



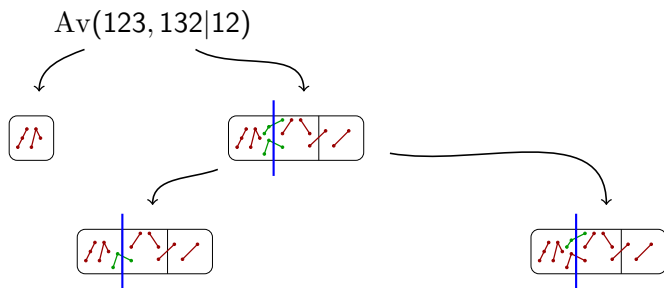
Disambiguation to combinatorial specification



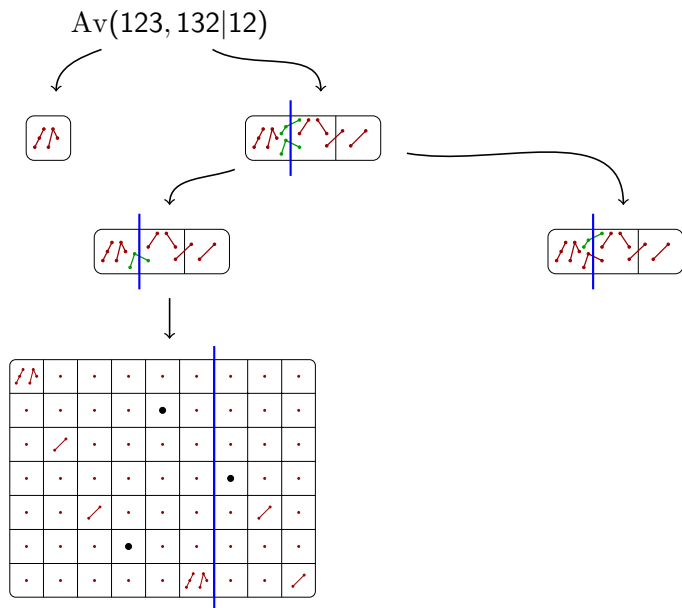
Disambiguation to combinatorial specification



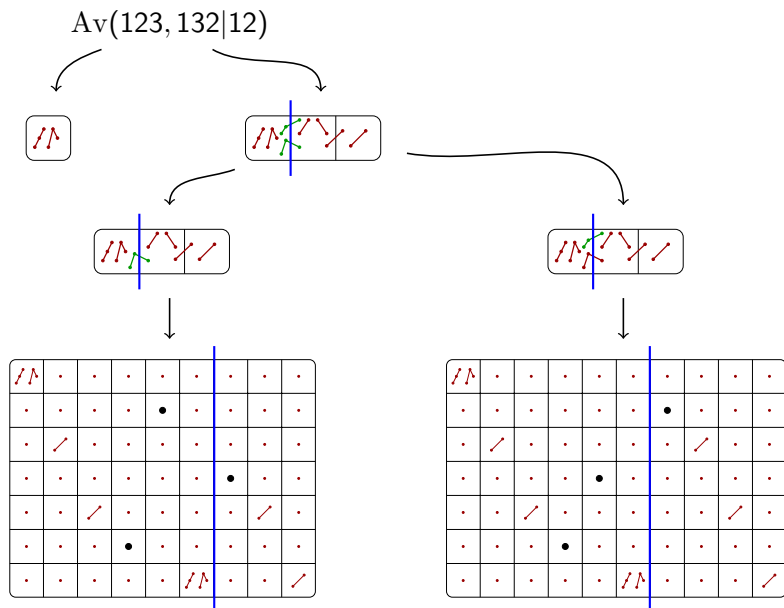
Disambiguation to combinatorial specification



Disambiguation to combinatorial specification



Disambiguation to combinatorial specification



$$F_0(x) = F_1(x) + F_2(x)$$

$$F_1(x) = \frac{1-x}{1-2x}$$

$$F_2(x) = F_3(x) + F_4(x)$$

$$F_3(x) = \frac{x^3}{(1-x)^2(1-2x)} \cdot F_1(x) \cdot F_1\left(\frac{x}{1-x}\right)$$

$$F_4(x) = \frac{x^3}{(1-x)(1-2x)^2} \cdot F_1(x) \cdot F_1\left(\frac{x}{1-x}\right)$$

Solving this system of equations gives us the generating function for the class is

$$F_0(x) = \frac{3x^4 - 15x^3 + 17x^2 - 7x + 1}{12x^4 - 28x^3 + 23x^2 - 8x + 1}$$

Results

We have ran the TileScope algorithm on all grid classes of the form $A \vee(A|B)$ where A and B are subsets of $\mathcal{S}_2 \cup \mathcal{S}_3$. We only considered grid classes which are lexicographically minimum down to symmetry and where at least one side is not a finite class, in total there are 1100 grid classes we considered.

$n m$	All	Success
1 1	20	7
2 1	72	27
2 2	87	74
3 1	80	36
3 2	172	171
4 1	60	27
5 1	24	15
6 1	4	3
<i>other</i>	581	581
Total	1100	941

Notable successes

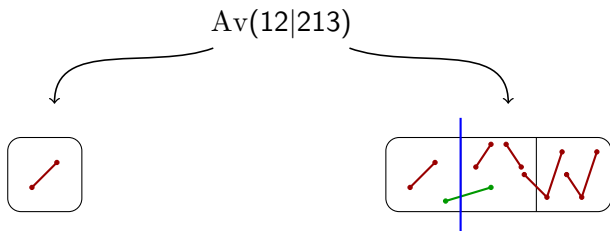
We have managed to automatically enumerate all* of the juxtapositions from Brignall and Sliacan [1].

Juxtaposition	Minimal polynomial
$A_V(12 312)$ $A_V(12 321)$	$-1 + x \cdot (x - 2) \cdot F(x)^2 + (x + 1) \cdot F(x)$
$A_V(12 213)$ $A_V(12 231)$	$x \cdot (x - 1)^2 \cdot F(x)^4 - (x - 1)^2 \cdot F(x)^3 + (3 \cdot x - 2) \cdot (x - 1) \cdot F(x)^2 + F(x) \cdot (x - 1) + x$
$A_V(12 132)$ $A_V(12 123)^*$	$(x - 1)^2 \cdot x^5 \cdot F(x)^4 - 2 \cdot x^3 \cdot (4 \cdot x - 1) \cdot (x - 1)^2 \cdot F(x)^3 + x \cdot (x - 1) \cdot (2 \cdot x^4 + 15 \cdot x^3 - 28 \cdot x^2 + 10 \cdot x - 1) \cdot F(x)^2 - (4 \cdot x - 1) \cdot (x - 1) \cdot (2 \cdot x^3 - x^2 - 4 \cdot x + 1) \cdot F(x) + x^5 + 4 \cdot x^4 - 21 \cdot x^3 + 25 \cdot x^2 - 9 \cdot x + 1$

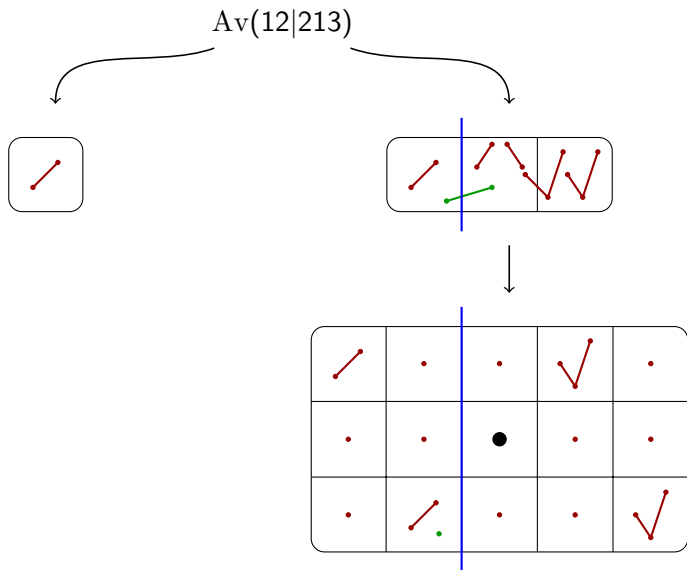
$Av(12|213)$

$Av(12|213)$ 

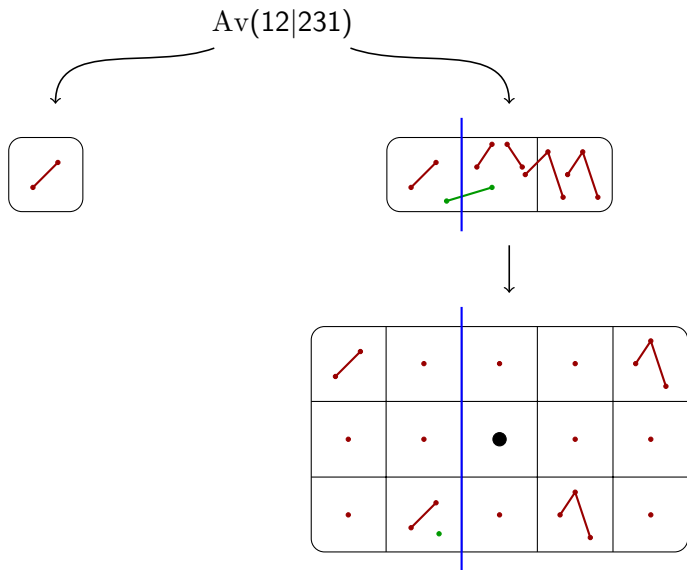
$Av(12|213)$



$Av(12|213)$



$Av(12|231)$



Non regular insertion encoding

$n m$	All	Success
1 1	18	5
2 1	56	11
2 2	15	5
3 1	60	16
3 2	16	15
4 1	45	12
4 2	10	10
5 1	18	9
5 2	2	2
6 1	3	2
Total	243	87

$n m$	All	Success
1 1	18	7
2 1	56	41
2 2	15	5
3 1	60	42
3 2	16	15
4 1	45	32
4 2	10	10
5 1	18	16
5 2	2	2
6 1	3	3
Total	243	173

Does there exist a juxtaposition of two rational classes which leads to a non-rational generating function?

Open questions

Does there exist a juxtaposition of two rational classes which leads to a non-rational generating function?

Can the method disambiguation for $1 \times N$ grid classes be generalized to other shapes of grid classes?

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If you have any interesting juxtapositions or $1 \times N$ grid classes you can fill out the form at <http://bit.ly/basisrequests>